AP Calculus BC
Chapter 10 Part 1– AP Exam Problems

<u>All problems are NO CALCULATOR unless otherwise indicated.</u>

<u>Parametric Curves and Derivatives</u>

1. In the *xy*-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$ for $-3 \le t \le 3$, is a line segment with slope.

   A) $\frac{3}{5}$ B) $\frac{5}{3}$ C) 3 D) 5 E) 13

2. If $x = t^3 - t$ and $y = \sqrt{3t+1}$, then $\frac{dy}{dx}$ at $t = 1$ is

   A) $\frac{1}{8}$ B) $\frac{3}{8}$ C) $\frac{3}{4}$ D) $\frac{8}{3}$ E) 13

3. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is:

   A) $y = 2x$ C) $y = 2x - 1$ E) $y = 8x + 13$
   B) $y = 8x$ D) $y = 4x - 5$

4. Consider the curve in the *xy*-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \ge 0$. The slope of the line tangent to the curve at the point where $x = 3$ is (Calculator)

   A) 20.086 B) 0.342 C) -0.005 D) -0.011 E) -0.033

5. If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2y}{dx^2} =$

   A) $\frac{3}{4t}$ B) $\frac{3}{2t}$ C) $3t$ D) $6t$ E) $\frac{3}{2}$

6. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

   A) $-\frac{5}{2}$ B) $-\frac{6}{5}$ C) 0 D) $\frac{4}{5}$ E) $\frac{6}{5}$

7. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

   A) $4e^{2t}\cos(2t)$ B) $\frac{e^{2t}}{\cos(2t)}$ C) $\frac{\sin(2t)}{2e^{2t}}$ D) $\frac{\cos(2t)}{2e^{2t}}$ E) $\frac{\cos(2t)}{e^{2t}}$

8. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

A) 0 B) 1 C) 0, $\frac{2}{3}$ D) 0, $\frac{2}{3}$, and 1 E) No value

9. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where t = 13, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

A) $-\frac{4}{3}$ B) $-\frac{3}{4}$ C) $-\frac{4\tan 13}{3}$ D) $-\frac{4}{3\tan 13}$ E) $-\frac{3}{4\tan 13}$

10. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

A) –1 B) 0 C) 2 D) –1, 2 E) –1, 0, 2

11. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point (–3, 8)?

A) $x = -3$ B) $x = 2$ C) $y = 8$ D) $y = -\frac{27}{10}(x+3)+8$ E) $y = 12(x+3)+8$

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12. (1973 BC4, p. 538 #33) A kite flies according to the parametric equations $x = \frac{t}{8}$ and $y = -\frac{3}{64}t(t-128)$ where t is measured in seconds and $0 < t \le 90$.

(a) How high is the kite above the ground at time t = 32 seconds?
(b) At what rate is the kite rising at t = 32 seconds?
(c) At what rate is the string being reeled out at t = 32 seconds?
(d) At what time does the kite start to lose altitude?

13. (1974 BC5) Given the parametric equations $x = 2(t - \sin t)$ and $y = 2(1 - \cos t)$.

(a) Find $\frac{dy}{dx}$ in terms of t.
(b) Find an equation of the line tangent to the graph at $t = \pi$.
(c) Find an equation of the line tangent to the graph at $t = 2\pi$.

14. (1982 BC6, p. 536 #9) Point $P(x, y)$ moves in the xy–plane in such a way that $\frac{dx}{dt}=\frac{1}{t+1}$ and $\frac{dy}{dt}=2t$ for $t\geq 0$.

(a) Find the coordinates of P in terms of t if, when $t = 1$, $x=\ln 2$ and $y = 0$.
(b) Write an equation expressing y in terms of x.
(c) Find the average rate of change of y with respect to x as t varies from 0 to 4.
(d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

15. (1984 BC2, p. 538 #32) The path of a particle is given for time $t > 0$ by the parametric equations $x=t+\frac{2}{t}$ and $y=3t^2$.

(a) Find the coordinates of each point on the path where the velocity of the particle in the x direction is zero.
(b) Find $\frac{dy}{dx}$ when $t = 1$.
(c) Find $\frac{d^2y}{dx^2}$ when $y = 12$.

16. (1989 BC4, p. 538 #30) Consider the curve given by the parametric equations $x=2t^3-3t^2$ and $y=t^3-12t$.

(a) In terms of t, find $\frac{dy}{dx}$.
(b) Write an equation for the line tangent to the curve at the point where $t=-1$.
(c) Find the x and y coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

17. (1993 BC2) The position of a particle at any time $t\geq 0$ is given by $x(t)=t^2-3$ and $y(t)=\frac{2}{3}t^3$. Find $\frac{dy}{dx}$ as a function of x.

Length of Parametric Curves

18. The length of the curve determined by the equations $x=t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

A) $\int_0^4 \sqrt{4t+1}\,dt$
B) $2\int_0^4 \sqrt{t^2+1}\,dt$
C) $\int_0^4 \sqrt{2t^2+1}\,dt$
D) $\int_0^4 \sqrt{4t^2+1}\,dt$
E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$

19. The length of the path described by the parametric equations $x=\cos^3 t$ and $y=\sin^3 t$, for $0\le t\le\frac{\pi}{2}$, is given by

A) $\int_0^{\frac{\pi}{2}}\sqrt{3\cos^2 t+3\sin^2 t}\,dt$

B) $\int_0^{\frac{\pi}{2}}\sqrt{-3\cos^2 t\sin t+3\sin^2 t\cos t}\,dt$

C) $\int_0^{\frac{\pi}{2}}\sqrt{9\cos^4 t+9\sin^4 t}\,dt$

D) $\int_0^{\frac{\pi}{2}}\sqrt{9\cos^4 t\sin^2 t+9\sin^4 t\cos^2 t}\,dt$

E) $\int_0^{\frac{\pi}{2}}\sqrt{\cos^6 t+\sin^6 t}\,dt$

20. The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0\le t\le 1$, is given by

A) $\int_0^1\sqrt{t^2+1}\,dt$

B) $\int_0^1\sqrt{t^2+t}\,dt$

C) $\int_0^1\sqrt{t^4+t^2}\,dt$

D) $\frac{1}{2}\int_0^1\sqrt{4+t^4}\,dt$

E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

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21. (1972 BC7) Let *C* be the curve defined from *t* = 0 to *t* = 6 by the parametric equations $x=\frac{t+2}{2},\quad y=t(6-t)$. Set up but do not evaluate an integral expression for the length of *C*.

22. (1974 BC5) Given the parametric equations $x=2(t-\sin t)$ and $y=2(1-\cos t)$. Set up but do not evaluate an integral expression representing the length of the curve over the interval $0\le t\le 2\pi$. Express the integrand as a function of *t*.

Vectors

23. If *f* is a vector-valued function defined by $f(t)=\left(e^{-t},\cos t\right)$, then $f''(t)=$

A) $-e^{-t}+\sin t$

B) $e^{-t}-\cos t$

C) $\left(-e^{-t},-\sin t\right)$

D) $\left(e^{-t},\cos t\right)$

E) $\left(e^{-t},-\cos t\right)$

24. A particle moves in the *xy*-plane so that at any time *t* its coordinates are $x=t^2-1$ and $y=t^4-2t^3$. At *t* = 1, its acceleration vector is

A) $(0,-1)$ B) $(0,12)$ C) $(2,-2)$ D) $(2,0)$ E) $(2,8)$

25. For any time $t \geq 0$, if the position of a particle in the *xy*-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is

A) $\left(2t, \dfrac{2}{2t+3}\right)$ C) $\left(2, \dfrac{4}{(2t+3)^2}\right)$ E) $\left(2, \dfrac{-4}{(2t+3)^2}\right)$

B) $\left(2t, \dfrac{-4}{(2t+3)^2}\right)$ D) $\left(2, \dfrac{2}{(2t+3)^2}\right)$

26. If a particle moves in the *xy*-plane so that at time *t* > 0 its position vector is $\left(\ln\left(t^2+2t\right), 2t^2\right)$, then at time *t* = 2, its velocity vector is

A) $\left(\dfrac{3}{4}, 8\right)$ B) $\left(\dfrac{3}{4}, 4\right)$ C) $\left(\dfrac{1}{8}, 8\right)$ D) $\left(\dfrac{1}{8}, 4\right)$ E) $\left(-\dfrac{5}{16}, 4\right)$

27. A particle moves on a plane curve so that at any time *t* > 0 its *x*-coordinate is $t^3 - t$ and its *y*-coordinate is $(2t-1)^3$. The acceleration vector of the particle at *t* = 1 is

A) (0, 1) B) (2, 3) C) (2, 6) D) (6, 12) E) (6, 24)

28. A particle moves in the *xy*-plane so that its position at any time *t* is given by $x = t^2$ and $y = \sin(4t)$. What is the speed of the particle when *t* = 3?

A) 2.909 B) 3.062 C) 6.884 D) 9.016 E) 47.393

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29. (1975 BC3) A particle moves on the circle $x^2 + y^2 = 1$ so that at time $t \geq 0$ the position is given by the vector $\left(\dfrac{1-t^2}{1+t^2}, \dfrac{2t}{1+t^2}\right)$.

(a) Find the velocity vector.
(b) Is the particle ever at rest? Justify your answer.
(c) Give the coordinates of the point that the particle approaches as *t* increases without bound.

30. (1987 BC5) The position of a particle moving in the *xy*-plane at any time *t*, $0 \leq t \leq 2\pi$, is given by the parametric equations $x = \sin t$ and $y = \cos(2t)$.

(a) Find the velocity vector for the particle at anytime *t*, $0 \leq t \leq 2\pi$.
(b) For what values of *t* is the particle at rest?
(c) Write an equation for the path of the particle in terms of *x* and *y* that does <u>not</u> involve trigonometric functions.

31. (1992 BC3) At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the *xy*-plane is given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

(a) Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

(b) Find the speed of the particle when $t = 1$.

(c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

32. (1993 BC2) The position of a particle at any time $t \ge 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at $t = 5$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

33. (1994 BC3) A particle moves along the graph of $y = \cos x$ so that *x*-component of acceleration is always 2. At time $t = 0$, the particle is at the point $(\pi, -1)$ and the velocity vector of the particle is (0, 0).

(a) Find the x and y coordinates of the position of the particle in terms of t.

(b) Find the speed of the particle when its position is $(4, \cos 4)$.

34. (1995 BC1) Two particles move in the xy – plane. For time $t \ge 0$, the position of particle *A* is given by $x = t - 2$ and $y = (t-2)^2$, and the position of particle *B* is given by $x = \frac{3t}{2} - 4$ and $y = \frac{3t}{2} - 2$.

(a) Find the velocity vector for each particle at time $t = 3$.

(b) Set up an integral expression that gives the distance traveled by particle *A* from $t = 0$ to $t = 3$. Do not evaluate.

(c) Determine the exact time at which the particles collide; that is, when the particles are at the same point at the same time. Justify your answer.

35. (1997 BC1) During the time period from $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.

(a) Find the position of the particle when $t = 2.5$.

(b) On a set of x and y-axes, sketch the graph of the path of the particle from $t = 0$ to $t = 6$. Indicate the direction of the particle along this path.

(c) How many times does the particle pass through the point found in part (a)?

(d) Find the velocity vector for the particle at any time t.

(e) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance the particle travels from time $t = 1.25$ to $t = 1.75$.

36. (1999 BC1) A particle moves in the *xy*-plane so that its position at any time *t*, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1+t)$ and $y(t) = 3\sin t$.

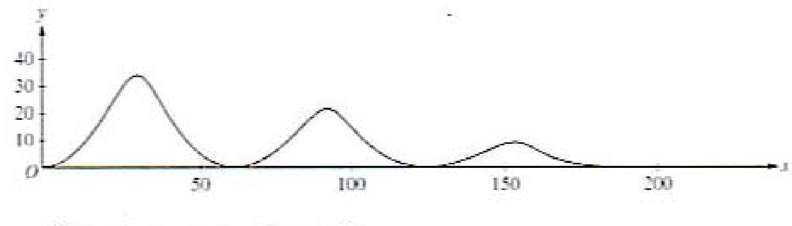
(a) Sketch the path of the particle on a set of *x* and *y*-axes. Indicate the direction of motion along the path.
(b) At what time *t*, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t),\ y(t))$ of the particle at this time?
(c) At what time *t*, $0 \le t \le \pi$, is the particle on the *y*-axis? Find the speed and the acceleration vector of the particle at this time.

37. (2000 BC4) A moving particle has position $(x(t),\ y(t))$ at time *t*. The position of the particle at time *t* = 1 is (2, 6), and the velocity vector at any time t > 0 is given by $\left(1 - \frac{1}{t^2},\ 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time *t* = 3.
(b) Find the position of the particle at time *t* = 3.
(c) For what time *t* > 0 does the line tangent to the path of the particle at $(x(t),\ y(t))$ have a slope of 8?
(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

38. (2001 BC1) An object moving along a curve in the *xy*-plane has position $(x(t),\ y(t))$ at time *t* with $\frac{dx}{dt} = \cos\left(t^3\right)$ and $\frac{dy}{dt} = 3\sin\left(t^2\right)$ for $0 \le t \le 3$. At time *t* = 2, the object is at position (4, 5).

(a) Write an equation for the line tangent to the curve at (4, 5).
(b) Find the speed of the object at time *t* = 2.
(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.
(d) Find the position of the object at time *t* = 3.

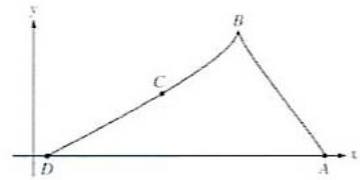


39. (2002 BC3) The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x = 10t + 4\sin t, \quad y = (20 - t)(1 - \cos t)$, where x and y are measured in meters. The derivatives of these functions are given by
$x' = 10 + 4\cos t, \quad y' = (20 - t)\sin t + \cos t - 1$.

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.
(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.
(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.
(d) For $0<t<18$, there are two times at which the car is a ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

40. (2002B BC1) A particle moves in the xy-plane so that its position at any time t, $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

(a) Sketch the path of the particle on a set of x and y-axes. Indicate the direction of motion along the path.
(b) Find the range of $x(t)$ and the range of $y(t)$.
(c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time?
(d) Is the distance traveled by the particle in $-\pi \le t \le \pi$ greater than 5π? Justify your answer.



41. (2003 BC2) A particle starts at point A on the positive x – axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t, where $x' = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y' = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x – axis.

(a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.
(b) The slope of the curve is undefined at point B. At what time t is the particle at B?
(c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.
(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

42. (2003B BC4) A particle moves in the xy – plane so that the position of the particle at any time t is given by $x(t) = 2e^{3t} + e^{-7t}$ and $y(t) = 3e^{3t} - e^{-2t}$.

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.
(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t\to\infty}\frac{dy}{dx}$.
(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.
(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

43. (2004 BC3) An object moving along a curve in the *xy*-plane has position $\left(x(t),\ y(t)\right)$ at time $t \geq 0$ with $\dfrac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\dfrac{dy}{dx}$ is not explicitly given. At time $t = 2$, the object is at position (1, 8).

(a) Find the *x*-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\dfrac{dy}{dt} = -7$. Write an equation for the line tangent to the curve at the point $\left(x(2),\ y(2)\right)$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $\left(x(t),\ y(t)\right)$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

44. (2004B BC1) A particle moving along a curve in the plane has position $(x(t), y(t))$ at time *t*, where $\dfrac{dx}{dt} = \sqrt{t^4 + 9}$ and $\dfrac{dy}{dt} = 2e^t + 5e^{-t}$ for all real values of *t*. At time $t = 0$, the particle is at the point (4, 1).

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(d) Find the x – coordinate of the position of the particle at time $t = 3$.

45. (2005B BC1) A particle moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time $t \geq 0$, where $\dfrac{dx}{dt} = 12t - 3t^2$ and $\dfrac{dy}{dt} = \ln\left(1 + (t-4)^4\right)$. At time $t = 0$, the particle is at the point (–13, 5). At $t = 2$, the object is at point *P* with x – coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y – coordinate of *P*.

(c) Write an equation for the line tangent to the curve at *P*.

(d) For what value of *t*, if any, is the object at rest? Explain your reasoning.

46. (2006 BC3) An object moving along a curve in the *xy*-plane is at position $(x(t), y(t))$ at time *t*, where $\dfrac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right)$ and $\dfrac{dy}{dt} = \dfrac{4t}{1 + t^3}$ for $t > 0$. At time $t = 2$, the object is at the point $(6, -3)$.

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time *t* is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of *t* and use it to evaluate $\lim\limits_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, and expression involving an improper integral that represents this value *c*.

47. (2006B BC2) An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where $\frac{dx}{dt} = \tan\left(e^{-t}\right)$ and $\frac{dy}{dt} = \sec\left(e^{-t}\right)$ for $t \geq 0$. At time $t = 1$, the object is at the point $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.
(b) Find the acceleration vector and the speed of the object at time $t = 1$.
(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.
(d) Is there a time $t \geq 0$ at which the object is on the y – axis? Explain why or why not.

48. (2007B BC2) An object moving along a curve in the xy – plane is at position $\left(x(t), y(t)\right)$ at time t with $\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right)$ and $\frac{dy}{dt} = \ln\left(t^2+1\right)$ for $t \geq 0$. At time $t = 0$, the object is at position $(-3, 4)$.

(a) Find the speed of the object at time $t = 4$.
(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.
(c) Find $x(4)$.
(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

49. (2008B BC1) A particle moving along a curve in the xy – plane has position $\left(x(t), y(t)\right)$ at time $t \geq 0$ with $\frac{dx}{dt} = \sqrt{3t}$ and $\frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$. The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.
(b) Find the y – coordinate of the position of the particle at time $t = 0$.
(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5?
(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

Answers

Parametric Curves and Derivatives

1. A	1998	BC	#2	92%
2. B	1985	BC	#30	73%
3. C	1988	BC	#34	68%
4. D	1993	BC	#25	54%
5. A	1993	BC	#6	39%
6. B	1993	BC	#4	75%
7. E	1997	BC	#2	92%
8. C	1997	BC	#18	62%
9. D	2003	BC	#4	79%
10. C	2003	BC	#7	29%
11. A	2003	BC	#17	36%

Length of Parametric Curves

18. D	1993	BC	#23	83%
19. D	1997	BC	#15	66%
20. C	1998	BC	#21	73%

Vectors

23. E	1998	BC	#77	87%
24. D	1985	BC	#4	69%
25. E	1988	BC	#15	75%
26. A	1993	BC	#28	86%
27. E	1998	BC	#10	75%
28. C	2003	BC	#84	62%